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Research Article

An Application of Interacting Boson Fermion-Fermion Model (IBFFM) for ^{134}Cs Nucleus

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Abstract

This work concerns the calculations of interacting boson fermion-fermion model (IBFFM) for the odd-odd nucleus ^{134}Cs. The energy levels (positive and negative parity states), electric transition probability B(E2), magnetic transition probability B(M1), quadrupole and magnetic dipole moments have been studied in this work. The IBFFM results are compared with the available experimental data. In the present work, the IBFFM pattern of total and parametric dependent level densities for the odd-odd nucleus ^{134}Cs is investigated and compared to the pattern found in previous investigations in the framework of combinatorial and spectral distribution approaches. When comparing the theoretical values with the available experimental values, it was found that there is a good match between them. This is due to the values of the Hamiltonian parameters that were found accurately, so this IBFFM model is considered one of the effective models in studying the nuclear structure of odd-odd nuclei. The level density of the odd-odd nucleus ^{196}Au is investigated in the interacting boson-fermion-fermion model (IBFFM) which accounts for collectivity and complex interaction between quasiparticle and collective modes. The IBFFM spin-dependent level densities show high-spin reduction with respect to Bethe formula. This can be well accounted for by a modified spin-dependent level density formula.

Keywords

IBFFM, Energy Levels, Electromagnetic Transitions

1. Introduction

A difficulty for theoretical investigations is the available data on the nucleus $^{134}_{55}\mathrm{Cs}_{79}$, which has been the subject of intensive experimental investigation [1-3].

The interacting boson model has been widely used to describe even-even and odd-even nuclei throughout the past ten years [4, 5]. This strategy was just recently expanded to odd-odd nuclei [6-10]. The IBFFM is a new model for odd-odd nuclei that was presented [6-8]. In a related advance, a few dynamical symmetries and supersymmetries, including odd-odd nuclei as well, have been created [8-13].

2. The Interacting Boson Fermion-Fermion Model (IBFFM)

The Interacting Boson Fermion-Fermion Model (IBFFM) is an extension of the Interacting Boson-Fermion Model (IBFM) where the Interacting Boson Model (IBM) core is connected to two fermions. The Hamiltonian of IBFFM is

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given by [6]:

$$H^{\wedge}_{IBFFM} = H^{\wedge}_{IBFM}(\pi) + H^{\wedge}_{IBFM}(\nu) + H^{\wedge}_{RES}(\pi\nu) - H^{\wedge}_{IBM} \quad (1)$$

For odd-even nuclei containing an odd proton and odd neutron, the IBFM Hamiltonians are indicated by the symbols $H_{IBFM}(\pi)$ and $H_{IBFM}(\nu)$ [5]. The IBM Hamiltonian is represented by H_{IBM} [4]; the H_{RES} stands for the residual proton-neutron interaction. The IBFFM code includes the residual interactions spin-spin, multipole-multipole, spin-spin-delta, and tensor interaction, which are represented by the study of Paar et al., [9]:

$$\mathrm{H}^{\wedge\,\pi\nu}_{\mathrm{RES}} = \mathrm{H}^{\wedge}_{\delta} + \mathrm{H}^{\wedge}_{\sigma\sigma\sigma} + \mathrm{H}^{\wedge}_{\sigma\sigma} + \mathrm{H}^{\wedge}_{\mathrm{T}} + \mathrm{H}^{\wedge}_{\mathrm{MM}} \quad (2)$$

where

$$\mathrm{H}^{\wedge}_{\delta} = 4\pi \mathrm{V}_{\delta}(\vec{r_{\pi}} - \vec{r_{\nu}})\ \delta\ ({}_{\pi}r - R_0)\ \delta(r_{\nu} - R_0) \quad (3)$$

is a surface delta-function interaction between the proton and neutron,

$$\mathrm{H}^{\wedge}_{\sigma\sigma\sigma} = 4\pi \mathrm{V}_{\sigma\sigma\sigma}(\vec{r_{\pi}} - \vec{r_{\nu}})(\vec{\sigma_{\pi}}\,.\,\vec{\sigma_{\nu}})\ \delta\ ({}_{\pi}r - R_0)\ \delta(r_{\nu} - R_0) \quad (4)$$

is a surface contact spin-spin interaction,

$$\mathrm{H}^{\wedge}_{\sigma\sigma} = -\sqrt{3}\mathrm{V}_{\sigma\sigma}\vec{\sigma_{\pi}}\,.\,\vec{\sigma_{\nu}} \quad (5)$$

is a spin-spin interaction,

$$\mathrm{H}^{\wedge}_{\mathrm{T}} = \mathrm{V}_{\mathrm{T}}\left[3\frac{(\vec{\sigma_{\pi}}\vec{r_{\pi}}).\,(\vec{\sigma_{\nu}}\vec{r_{\nu}})}{r^2_{\pi\nu}} - \vec{\sigma_{\pi}}\,.\,\vec{\sigma_{\nu}}\right] \quad (6)$$

is a tensor interaction, and

$$\mathrm{H}^{\wedge}_{\mathrm{MM}} = 4\pi\frac{\delta(r_{\pi} - r_{\nu})}{r_{\pi}r_{\nu}}\sum_{km}\mathrm{V_k}\,\mathrm{Y}^{*}_{\mathrm{km}}(\theta_{\pi},\varphi_{\pi})\ \mathrm{Y}_{\mathrm{km}}(\theta_{\nu},\varphi_{\nu}) \quad (7)$$

the residual interaction Hamiltonian was calculated as follows:

$$\mathrm{H}^{\wedge}_{\mathrm{RES}} = 4\pi V_{\delta}\delta(\mathrm{r}_{\pi}\text{-}\mathrm{r}_{\nu})\delta(\mathrm{r}_{\pi}\text{-}\mathrm{R}_0)\text{-}\sqrt{3}\mathrm{V}_{\sigma\sigma}(\sigma_{\pi}.\sigma_{\nu}) + \mathrm{V}_{\mathrm{T}}\left[3\frac{(\vec{\sigma_{\pi}}\vec{r_{\pi}}).\,(\vec{\sigma_{\nu}}\vec{r_{\nu}})}{r^2_{\pi\nu}} - \vec{\sigma_{\pi}}\,.\,\vec{\sigma_{\nu}}\right] \quad (8)$$

where $R_0 = 1.2\ \mathrm{A}^{1/3}$, the parameters V_{δ}, $\mathrm{V}_{\sigma\sigma\sigma}$, $\mathrm{V}_{\sigma\sigma}$, $\mathrm{V_T}$ and $\mathrm{V_R}$ are constants, and determined the strengths of the interactions.

IBFFM computation for ^{134}Cs was performed by coupling valence-shell proton and neutron quasiparticles to the IBM boson core.

3. Results and Discussion

3.1. Energy Spectra

The boson core fit the even-even nucleus in the transitional limit (O(6) limit) with the following parameters: A= 0.7, B = 0.8, C= 0.01, and N = 4 (total number of bosons). This parameters shows a $\gamma -$dependence on the potential energy surface, in qualitative accord with earlier computations for $Z > 50$, $N < 82$ even-even nuclei carried out in the classic collective model [14-16]. The low-lying portion of the identical spectrum of ^{132}Xe was replicated in the second phase using the renormalized O(6) parameters and the boson space truncated to N = 2. This was done in order to minimize the scope of computations. The low-lying portion of the identical spectrum of ^{132}Xe was replicated in the second phase using the renormalized O(6) parameters and N = 2 is the truncated boson space. This was done in order to minimize the scope of calculations.

The appearance of dynamical symmetries linked to the O(6) limit is related to this property [17]. For the SU(3) limit, a comparable circumstance can be found [18]. The estimate for ^{134}Cs includes the low-lying valence-shell quasiparticle states corresponding to the adjacent odd-even nuclei, ^{133}Xe and ^{133}Cs. These are quasi-protons ($g_{7/2}$, $d_{5/2}$) and quasi-neutrons ($d_{3/2}$, $d_{5/2}$, $s_{1/2}$). The energies of quasiparticle for the protons $g_{7/2}$ and $d_{5/2}$ are 0 and 0.7 MeV, respectively with probability of occupation is 0.446 and 0.166.

The quasiparticle energies for $d_{3/2}$, $d_{5/2}$, and $s_{1/2}$ neutrons are 0, 0.7, and 0.9 MeV, with corresponding occupation probabilities of 0.785, 0.988, and 0.969 respectively. The occupancy probabilities are calculated using the BCS parameter proposed by Kisshnger-Sorensen [19]. Conversely, the energy levels of approximately ^{134}Cs are fitted with three quasiparticle energy spacings. This fits with earlier discussions of odd-odd nuclei in the context of the boson-fermion method [20]. Specifically, the quasiparticle energy normalization is caused by the boson-fermion interaction, which results in relative shifts of the quasi-proton quasi-neutron multiplets. As a result, the effective values used to describe odd-odd nuclei may differ from the experimental quasi-particle energies from the neighboring odd-even nuclei.

In $^{134}_{55}\mathrm{Cs}_{79}$, the four lowest-lying positive-party levels are 4^+, 5^+, 3^+, and the state at 0.174 MeV, which is designated as 2^+ or 3^+. In the (d,p) reaction, these states are significantly stimulated by $L=2$ transfer, and in the (t, α) reaction, by $L=4$ transfer. Assuming these four levels contain sizable components with $(\pi g_{7/2}, \nu\, d_{3/2})$ and the 2^+ member of the $(\pi g_{7/2}, \nu d_{3/2})$ multiplets, this is consistent with the $(\pi g_{7/2}, \nu\, d_{3/2})$ assumption [21]. We have examined in IBFFM how parameters affects the multiplet 2^+, 3^+, 4^+, and 5^+ splitting that is computed.

If the dynamical and exchange interaction strengths are relatively small ($\Gamma_0^\pi, \Gamma_0^\nu \le 0.3$, $\Lambda_0^\pi, \Lambda_0^\nu \le 1$), the estimated quadrupole moment of the 4^+ ground state has a negative sign, which is in contradiction with experiment [2].

It can be observed from examining the four-dimensional parameter space ($\Gamma_0^\pi, \Gamma_0^\nu$, $\Lambda_0^\pi, \Lambda_0^\nu$) that this pattern is comparatively stable within that range of values. However, it is important to notice that for $\nu^2(g_{7/2}) < 0.5$, this pattern is consistent with the parabolic rule [21-23] prediction.

However, by further increasing the parameter strengths for $\Lambda_0^\pi, \Lambda_0^\nu \ge 1.5$, the states 4^+, 5^+, 3^+, and 2^+ (lie on a parabola) open up and become the domain of parametrization for which we can obtain the experimentally observed pattern, with the 4^+ state acting as the ground state with positive quadrupole moment. Table 1 displays the calculated positive-parity levels of ^{134}Cs up to 0.300 MeV.

The computation employed the following boson-fermion interaction parameters: $\Gamma_0^\pi = 1.2$ MeV, $\Gamma_0^\nu = 0.42$ MeV, $\Lambda_0^\pi = 2.2$ MeV, $\Lambda_0^\nu = 2.2$ MeV, $V_{\sigma\sigma} = 1.2$ MeV, $V_\delta = -0.13$ MeV. The 4^+, 5^+, 3^+, and 2^+ states wave functions are dominated by components that have $\pi\, g_{7/2}$ and $\nu\, d_{5/2}$ configurations.

Including $\nu\, h_{11/2}$, the quasiparticle with negative panty, additionally, we have determined the occupation probability for negative-parity states using Kisslinger-Sorensen (v^2=0.850), attempting to replicate the intricate experimental pattern that would indicate a deformed parabola opening up.

With the exception of $\Gamma_0^\nu = 0.82$ MeV, $\chi = -2.2$ MeV, we utilize the same boson-fermion parametrization as for the positive-parity states to get the negative-parity levels depicted in Table 1 up to 0.600 MeV.

Table 1. IBFFM calculations and experimental data [24] for low-lying positive parity energy states for ^{134}Cs nucleus (in MeV units).

Positive parity states			**Negative parity states**		
Levels	**Exp. [24]**	**IBFFM**	**Levels**	**Exp. [24]**	**IBFFM**
4_1^+	0.0	0.0	8_1^-	0.138	0.142
5_1^+	0.0112	0.010	3_1^-	0.1764	0.170
3_1^+	0.0600	0.077	4_1^-	0.193	0.211
3_2^+	0.173	0.167	6_1^-	0.257	0.289
1_1^+	0.176	0.180	4_2^-	0.267	0.271
3_3^+	0.190	0.195	7_1^-	0.344	0.376
2_1^+	0.197	0210	6_2^-	0.382	0.391
5_2^+	0.209	0.218	6_3^-	0.434	0.442

Positive parity states			Negative parity states		
Levels	Exp. [24]	IBFFM	Levels	Exp. [24]	IBFFM
3_4^+	0.234	0.239	5_1^-	0.450	0.481
2_2^+	0.274	0.281	4_3^-	0.483	0.510
2_3^+	0.290	0.301	4_4^-	0.570	0.589
4_2^+	0.377	0.367	5_2^-	0.613	0.633
3_5^+	0.451	0.433	5_3^-	0.624	0.645
4_3^+	0.454	0.476	4_3^-	0.643	0.681
3_6^+	0.502	0.511	2_1^-	0.684	0.697
4_3^+	0.519	0.522	3_2^-	0.701	0.721
5_3^+	0.539	0.559	2_2^-	0.715	0.20
2_4^+	0.579	0.588	4_4^-	0.752	0.755
2_5^+	0.584	0.611	6_4^-	0.783	0.792

It should be emphasized that there are certain uncertainties in the experimental data about the occupation of $g_{9/2}$, such as whether $v^2(g_{7/2})$ is marginally over or under 0.52 [20]. As a result, we looked into potential parameterizations with $v^2(g_{7/2}) >$ 0.52 in IBFFM. Nevertheless, in this instance, we were unable to replicate the characteristics of the experimentally observed low-lying positive and negative-parity multiplets. The 4_1^+ state wave function, which was produced by diagonalizing H_{IBFFM} in basis $|(j_\pi, j_\nu)\, J_{\pi\nu}; n_d R_d; I\rangle$, has the following biggest components (see Table 2):

Table 2. The wave function components of some of the lowest lying levels in ^{134}Cs nucleus as calculated in the IBFFM model.

levels	Wavefunction components
4_1^+	$0.57\|(\pi g_{7/2}\nu d_{3/2})4,00;4\rangle+0.74\|)\pi g_{7/2},\nu d_{3/2})5,12;4\rangle-0.13\|(\pi gh_{7/2},\nu d_{3/2})4;20;4\rangle+...$
8_1^-	$0.593\|(\pi g_{7/2},\nu h_{11/2})8;00;8\rangle\ +0.48\|(g_{7/2},\nu h_{11/2})8,12;8\rangle\ -0.26\|(\pi g_{7/2},\nu h_{11/2})8,20;8\rangle+...$

The wavefunctions of IBFFM do not show a single component, as would be expected for such a complex nucleus, indicating that assuming simply a one-quasi-proton-one quasi-neutron configuration as a zeroth-order approximation would be overly simplistic. Interestingly, though, we get the low-lying states that one would anticipate based on the zeroth-order approximation ($(\pi g_{7/2},\nu d_{3/2})$. even if the wave functions are more complicated, 2^+, 3^+, 4^+, 5^+, and $(\pi g_{7/2},\nu h_{11/2})$ $2^-, 3^-, ..., 9^-$. Furthermore, many characteristics of these states can be roughly explained within the context of the zeroth-order classification and the simple parabolic rule [21-23].

3.2. Electromagnetic Properties

The eigenstates of the IBFFM-2 Hamiltonian enable us to determine the electric quadrupole (E2) and magnetic dipole (M1) properties of odd-odd nuclei [6].

$$T^{[E2]} = T_B^{[E2]} + T_F^{[E2]} \tag{9}$$

the boson operator is written as [6]:

$$T_B^{[E2]} = e_B Q_B^{\wedge} \tag{10}$$

where e_B is the boson effective charge in (e. b) units and $Q_B^{\wedge}$ stands for the quadrupole operator is takes by [6]:

$$Q_\rho^{[2]} = \left[\left(d^+ \times s^{\sim}\right)_\rho^{[2]} + \left(s^+ \times d^{\sim}\right)_\rho^{[2]}\right]^{[2]} + \chi\left[d^+ \times d^{\sim}\right]_\rho^{[2]} \tag{11}$$

the fermion E2 operator adopts the form [6]:

$$T_F^{[E2]} = -e_F \sum_{jj'} \frac{1}{\sqrt{5}} \gamma_{jj'} \left[a^+ \times a_{j'}^{\sim}\right]^{[2]} \tag{12}$$

with e_F standing for the effective fermion charge, e_B is fitted to reproduced the experimental $B(E2;\ 2_1^+ \to 0_1^+)$ value of the corresponding even-even boson-core nuclei, as in earlier research [12, 13], whereas e_F is assumed to be equal to e_B for all the investigated odd-mass nuclei. The quadrupole moments equation is given by [6, 17]:

$$Q_J = \left(\frac{16\pi}{5}\right)^{1/2} \left\langle J \left\| T^{(E2)} \right\| J \right\rangle \sqrt{J(2J+1)/\{(2J+1)(J+1)(2J+3)\}} \tag{13}$$

where the reduced electric transition probability $B(E2)$ value is given as [17]:

$$B(E2; J_i^+ \to J_f^+) = \frac{1}{2J_i+1} \left| < J_f^+ \left\| T^{(E2)} \right\| J_i^+ > \right|^2 \tag{14}$$

The magnetic M1 transition operator is given by the formula [12]:

$$T^{[M1]} = \sqrt{\frac{3}{4\pi}} \left(T_B^{\wedge[M1]} + T_F^{\wedge[M1]}\right) \tag{15}$$

The M1 boson operator is proportional to the angular momentum operator of boson $T_B^{\wedge[M1]} = g_B L^{\wedge}$ with the gyromagnetic factor $g_B = \mu_{2_1^+}/2$ given in terms of the magnetic moment $\mu_{2_1^+}$ of the 2_1^+ state of the even-even nucleus. The M1 operator for fermion part is given by [12, 17]:

$$T_F^{\wedge[M1]} = \sum_{jj'} g_{jj'} \sqrt{\frac{j(j+1)(2j+1)}{3}} \left[a_j^+ \times a_{j'}^{\sim}\right]^{[1]} \tag{16}$$

Table 3 displays the computed moments of the electric quadrupole and magnetic dipole of the energy states corresponding to the lowest-lying positive and negative-parity multiplets.

The usual values for the effective proton and neutron charges are $e_\pi^{s\ p} = 1.5$ and $e_\nu^{s\ p} = 1.0$, respectively, whereas the boson charge, $e_{vib} = 3.8$, is aligned with the 4^+ state electric quadrupole moment. For medium-heavy nuclei, the boson charges range ($2 \le e_{vib} \le 4$) that was formerly employed is qualitatively consistent with this result. According to conventional values, the ratios of gyromagnets are $g_l^\pi = 1.0$, $g_l^\nu = 0.0$, $g_s^\pi = 0.6\ g_s^{\nu\ free}$, $g_s^\nu = 0.5\ g_s^{\nu\ free}$, $g_R = Z/A$. The concordance between the measured and estimated magnetic moment of the ground state is improved by a little decrease in g_s^ν. The table illustrates how successfully the g-values that were used to simulate the magnetic dipole moments of states 5^+ and 8^-.

Now let us discuss the sign of $Q(4_1^+)$. Upon nearer examination, the off-diagonal boson contributions of $\Delta n_d = 1$ are responsible for the major partial contributions to $Q(4_1^+) = 20$ e.b.

$$\left\langle (\pi g_{7/2}, \nu h_{3/2})4, 00; 4 \right| Q^{\wedge} \left| (\pi g_{7/2}, \nu h_{3/2})4, 12; 4 \right\rangle = 22 \text{ e.b}$$

$$\left\langle (\pi g_{7/2}, \nu h_{3/2})5,\ 12; 4 \right| Q^{\wedge} \left| (\pi g_{7/2}, \nu h_{3/2})5,\ 24; 4 \right\rangle = 0.10 \text{ e.b}$$

Table 3. *Electric quadrupole moments and Magnetic dipole moments for states in ^{134}Cs.*

levels	**Q(e.b).**		**μ(μ$_N$)**	
	Exp. [24]	**IBFFM**	**Exp. [24]**	**IBFFM**
4_1^+	0.39	0.389	2.99	2.83
5_1^+	-	0.044	2.33	3.33
3_1^+	-	-0.55	-	2.40
2_1^+	-	-0.39	-	1.99
8_1^-	-	1.33	1.10	1.18
3_1^-	-	0.97	-	-1.99
4_1^-	-	0.71	-	-0.98
6_1^-	-	0.55	-	0.47
5_1^-	-	0.45	-	-0.24
7_1^-	-	1.32	-	0.89

The big components relation (1) and the tiny components relation (2) are the off-diagonal matrix elements, respectively. The latter two tiny coefficients contribute 2.9% and 0.62%, respectively, to the 4_1^+ state wave function.

The computed and experiment ratios are compared in Table 4 between the multiplets with the lowest positive and negative parity. The reduced transition probabilities for the $5_1^+ \to 4_1^+$ transitions that have been calculated are:

$$B(E2;5_1^+ \to 4_1^+) = 0.189\ e^2\ b^2,$$

$$B(M1;5_1^+ \to 4_1^+) = 0.0035\ \mu_N^2$$

There is consistency between the huge $Q(4_1^+)$ and the high value of B(E2). The partial contributions incoherence is the cause of $B(M1;5_1^+ \to 4_1^+) = 0.0035\ \mu_N^2$ decrease.

The 5_1^+ state half-life, $T_{1/2}(5_1^+) = 40$ ns, is obtained by tabulating internal conversion coefficients and using the computed B (M 1) and B(E2) values for the $5_1^+ \to 4_1^+$ transition. This is in reasonable with experimental data of 45 ns.

With the help of the half-life of the 6_1^- state $T_{1/2}(6_1^-) = 20$ ns using the experimental branching ratio $I(6^- \to 5^-)/I(6^- \to 4^-)$ and the calculated internal conversion constants, this corresponds reasonably well with the experimental measurement of 12 ns. These values are calculated for the $B(E2;6^- \to 4^-)$ and $B(E2;6^- \to 8^-)$ transitions. As a result, the distribution of boson charges found from matches the available lifetimes [23, 24].

***Table 4**. Electromagnetic transition multiplet for ^{134}Cs nucleus.*

transitions	**Excitation Energy (MeV)**	I_γ^{rel} **Exp.**	I_γ^{rel} **IBFFM**
$5_1^+ \to 4_1^+$	0.112	1	1
$3_1^+ \to 4_1^+$	0.060	1	0.06
$3_1^+ \to 5_1^+$	0.048	< 0.05	0.03
$2_1^+ \to 3_1^+$	1.139	1	0.18
$2_1^+ \to 4_1^+$	1.738	0.01	1
$4_1^- \to 3_1^-$	0.173	1	1
$5_1^- \to 4_1^-$	0.741	1	0.9
$5_1^- \to 6_1^-$	0.106	-	0.10
$5_1^- \to 3_1^-$	0.931	-	0.008
$6_1^- \to 8_1^-$	1.184	1	1
$6_1^- \to 4_1^-$	0.653	0.03	0.061
$7_1^- \to 8_1^-$	2.056	1	0.92
$7_1^- \to 6_1^-$	0.873	0.23	0.29
$7_1^- \to 5_1^-$	0.767	-	0.009

4. Conclusion

In this work the nuclear structure and electromagnetic transition for ^{134}Cs nucleus within IBFFM model have been studied. Theoretically, the transitions between the levels for $|\Delta I| \le 2$, $\Delta\pi = 0$ have the strongest branches at $2_1^+ \to 3_1^+$, $3_1^+ \to 4_1^+$, $5_1^+ \to 4_1^+$ and $7_1^- \to 8_1^-$, $5_1^- \to 4_1^-$, $6_1^- \to 8_1^-$ respectively. The majority of the $|\Delta I| = 1$ transitions are M1 type, which is consistent with experimental results.

The single-particle values are similar to the calculated B(M1) values between the negative-parity states $B(M1;7_1^- \to 8_1^-) = 0.665\ \mu_N^2$, $B(M1;5_1^- \to 4_1^-) = 1.159\ \mu_N^2$.

Conflicts of Interest

The authors declare no conflicts of interest.

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