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Critical Point Symmetry, X (5), in ^{154}Gd

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Abstract: The positive-negative parity states, potential energy surfaces, V(β, γ), transition probabilities, B(E1), B(E2), staggering effect and electric monopole strength, X (E0/E2), values of ^{154}Gd have been calculated within the frame work of the interacting boson approximation model (I BA − 1). The results obtained are compared to the available experimental, theoretical data and reasonable agreement has achieved. The potential energy surfaces, levels energy and transition probability ratios show that ^{154}Gd is an X (5) candidate.

Keywords: Levels Energy, Transition Probability, B(E1), B(E2), Electric Monopole Strength, X (E0/E2)

1. Introduction

The even-even ^{154}Gd isotope has subjected to an extensive study in the past few years where the region of N= 90 shows rapid changes of nuclear deformation parameters as a function of the particle number. Many authors have studied this isotope experimentally and theoretically.

Experimentally [1-10] different techniques were applied. Silicon and germanium detector arrays, (α, 2n), (α, 4n) reactions, AFRODITE γ-ray spectrometer, combination of γ - ray scattering experiments and γ - γ coincidence following the electron capture of ^{154m}Tb were used. They detected levels energy, quadrupole moment, two neutron separation energies, B(E1), B(E2) values and magnetic moment.

Theoretically authors [11-22] have studied all the important probes for ^{154}Gd nucleus as energy spectra, moment of inertia, quadrupole moments, octupole and hexadecupole degrees of freedom, B(E1), B(E2),B(E3), magnetic moment and quadrupole moment using different theoretical approaches and models.

The aim of the present work is to use the IBA-1 model [23-26] for the following tasks:

1. Calculating the potential energy surfaces, V(β, γ);
2. Calculating levels energy, electromagnetic transition rates B(E1) and B(E2);
3. Studying the back bending;
4. Calculating staggering effect;
5. Detect any interactions between the +ve and –ve parity States and calculating the electric monopole strength X(E0/E2).

2. Levels Energy in IBA-1 Model

IBA-1 model was applied to the positive and negative parity states of ^{154}Gd isotope. The Hamiltonian employed in the present calculation is:

$$H = EPS.n_d + PAIR.(P.P) + \frac{1}{2}ELL.(L.L) + \frac{1}{2}QQ.(Q.Q) + 5OCT.(T_3.T_3) + 5HEX.(T_4.T_4) \quad (1)$$

where

$$P{\cdot}p = \frac{1}{2}\left[\left\{\left(s^\dagger s^\dagger\right)_0^{(0)} - \sqrt{5}\left(d^\dagger d^\dagger\right)_0^{(0)}\right\} x \left\{\left(ss\right)_0^{(0)} - \sqrt{5}\left(\tilde{d}\tilde{d}\right)_0^{(0)}\right\}\right]_0^{(0)} \quad (2)$$

$$L{\cdot}L = -10\sqrt{3}\left[\left(d^\dagger\tilde{d}\right)^{(1)} x \left(d^\dagger\tilde{d}\right)^{(1)}\right]_0^{(0)} \quad (3)$$

$$Q{\cdot}Q = \sqrt{5}\left[\left\{\left(S^\dagger\tilde{d} + d^\dagger s\right)^{(2)} - \frac{\sqrt{7}}{2}\left(d^\dagger\tilde{d}\right)^{(2)}\right\} x \left\{\left(s^\dagger\tilde{d} + \tilde{d}s\right)^{(2)} - \frac{\sqrt{7}}{2}\left(d^\dagger\tilde{d}\right)^{(2)}\right\}\right]_0^{(0)} \quad (4)$$

$$T_3{\cdot}T_3 = -\sqrt{7}\left[\left(d^\dagger\tilde{d}\right)^{(3)} x \left(d^\dagger\tilde{d}\right)^{(3)}\right]_0^{(0)} \quad (5)$$

$$T_4 \bullet T_4 = 3\left[\left(d^\dagger \tilde{d}\right)^{(4)} x \left(d^\dagger \tilde{d}\right)^{(4)}\right]_0^{(0)} \qquad (6)$$

In the previous formulas, n_d is the number of bosons; P.P, L.L, Q.Q, $T_3.T_3$ and $T_4.T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and PAIR, ELL, QQ, OCT, HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

3. Transition Rates

The electric quadrupole transition operator employed in this study is given by:

$$T^{(E2)} = E2SDx\left(s^{?}\tilde{d}+d\ s\right)^{(2)}+\frac{1}{\sqrt{5}}E2DDx\left(d\ \tilde{d}\right)^{(2)} \qquad (7)$$

where:

$T^{(E2)}$: absolute transition probability of the electric quadrupole (E2) transition, and E2SD and E2DD: adjustable parameters

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E2, I_i - I_f) = \left[< I_f \left\| T^{(E2)} \right\| I_i >\right]^2 \Big/ (2I_i + 1) \qquad (8)$$

where

I_i: the initial state of the electric quadrupole transition, and

I_f: the final state of the electric quadrupole transition.

4. Results and Discussion

4.1. The Potential Energy Surfaces

The potential energy surfaces, V(β, γ), for ^{154} Gd nucleus as a function of the deformation parameters β and γ has been calculated using [27] equation:

$$E\ N_\upsilon\ N_\P\ (\beta, \gamma) = < N_\P\ N_\upsilon; \beta\gamma \mid H_{np} \mid N_\P\ N_\upsilon; \beta\gamma >$$

$$= \varepsilon_d (N_\upsilon + N_\P)\beta^2(1+\beta^2)^{-1} +$$

$$\beta^2(1+\beta^2)^{-2}\ \{k\ N_\P\ N_\upsilon[4-(\bar{X}_\Pi + \bar{X}_\nu)\beta\cos 3\gamma +$$

$$\bar{X}_\Pi\ \bar{X}_\nu\ \beta^2] + N_\upsilon(N_\upsilon-1)\left(\frac{C_o}{10}+\frac{C_2}{7}\right)\beta^2\} \qquad (9)$$

where

$$\bar{X}_\rho = (2/7)^{0.5}\ X_\rho\ \ \rho = \P \text{ or } \upsilon \qquad (10)$$

The calculated potential energy surfaces, V (β, γ), are Presented in Figure 1. The deviation from spherical, an harmonic vibrator U (5), to rotational characters, SU (3), at the critical symmetry point has supported quite well as well as the energy and transition probability ratios the X(5) characters to ^{154}Gd nucleus, Table. 1.

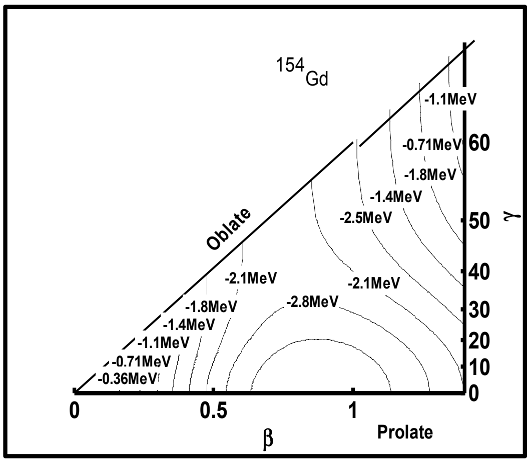

Figure 1. *Contour plot of the potential energy surfaces.*

4.2. Energy Spectra

The energy of the positive and negative parity states of ^{154} Gd isotope are calculated using computer code PHINT [28] with the parameters EPS = 0.424, P AIR = 0.000, ELL = 0.0084, QQ = −0.0244, OCT = 0.000, HEX = 0.000 all in MeV while, E2SD =0.1450(eb), E2DD = −0.4289(eb). A comparison between the experimental spectra [29] and our calculations, using values of the model parameters given in Table 1 for the ground state, β1, β2, γ1, γ2 and (−ve) parity bands are illustrated in Figure 2. The agreement between the calculated levels energy and their correspondence experimental values are fairly good in low-lying states and slightly higher for the higher excited states. We believe this is due to the change of the projection of the angular momentum which is due mainly to band crossing.

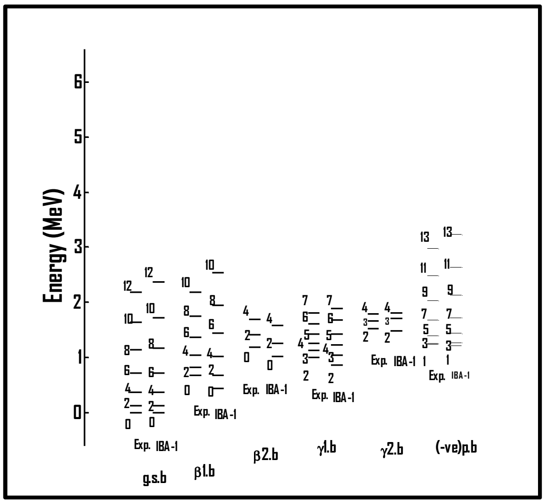

Figure 2. *Comparison between exp. [29] and theo. IBA calculations.*

4.3. Transition Rates

Unfortunately there is no enough measurements of electromagnetic transition rates B(E1) or B(E2) for ^{154} Gd nucleus. The only measured value is B(E2, $0^+_1 \rightarrow 2^+_1$) = 3.89(7) [30] which presented in Table 2. The parameters E2SD and E2DD are used in the computer code NPBEM [28] for calculating the electromagnetic transition rates and then normalized to the experimental value, B(E2, $0^+_1 \rightarrow 2^+_1$).

No new parameters are introduced for calculating electromagnetic transition rates B(E1) and B(E2) of intra band and inter band. It is clear that the electromagnetic transition of the γ transition within any band has a large value while it is small between intra bands which is due to either a mixture of M 1 or it is forbidden.

4.4. Back Bending

The moment of inertia J and energy parameters ω are calculated using Eq. (11, 12):

$$2J/\hbar^2 = (4I-2)/\Delta E(I \rightarrow I-2) \quad (11)$$

$$(\hbar\omega)^2 = (I^2 - I + 1)[\Delta E(I \rightarrow I-2)/(2I-1)] \quad (12)$$

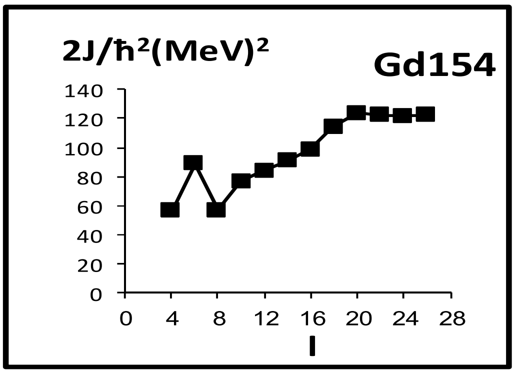

Figure 3. *Back bending.*

Table 1. *Energy and transition probability ratios.*

Nucleus	$E4_1^+/E2_1^+$	$E6_1^+/E2_1^+$	$E8_1^+/E2_1^+$	$E0_2^+/E2_1^+$	$E6_1^+/E0_2^+$	$E0_3^+/E2_1^+$	$BE2(4_1^+ \text{--} 2_1^+) / BE2(2_1^+ \text{--} 0_1^+)$
^{154}Gd	3.00	5.83	9.30	5.52	1.05	9.60	1.52
X(5)	3.02	5.83	9.29	5.65	1.53	6.03	1.58

Figure 3. shows forward bending for ^{154} Gd at $I^+ = 6$ and that bending has explained as due to partial rotational alignment of a pair of neutrons near the Fermi surface.

Table 2. *Calculated B(E2) and B(E1).*

I_i^+	I_f^+	B(E2) $_{Exp*.}$	B(E2) $_{IBA}$	I_i^+	I_f^+	B(E1) $_{Exp.}$	B(E1) $_{IBA}$
0_1	2_1	3.89(7)	3.8965	1_1	0_1	——	0.1087
2_2	0_1	——	0.0005	1_1	0_2	——	0.0786
2_2	0_2	——	0.4922	3_1	2_1	——	0.2465
2_3	0_1	——	0.0168	3_1	2_2	——	0.0826
2_3	0_2	——	0.0123	3_1	2_3	——	0.0083
2_3	0_3	——	0.0498	3_2	2_1	——	0.0352
2_4	03	——	0.3081	3_2	2_2	——	0.0214
2_4	0_4	——	0.0358	3_2	2_3	——	0.0564
2_2	2_1	——	0.0628	5_1	4_1	——	0.3710
2_3	2_1	——	0.0165	5_1	4_2	——	0.0733
2_3	2_2	——	0.4976	5_1	4_3	——	0.0043
4_1	2_1	——	1.1878	7_1	6_1	——	0.4943
4_1	2_2	——	0.0694	7_1	6_2	——	0.0631
6_1	4_1	——	1.3454	9_1	8_1	——	0.6193
6_1	4_2	——	0.0587	9_1	8_2	——	0.0545
8_1	6_1	——	1.4041	11_1	10_1	——	0.7469
8_1	6_2	——	0.0467				
8_1	6_3	——	0.0234				
10_1	8_1	——	1.3954				

Ref. [30]

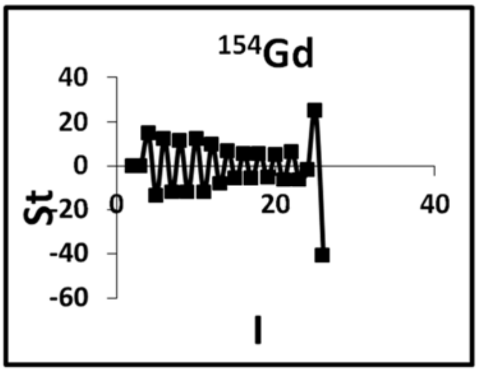

Figure 4. *Beat pattern.*

4.5. The Staggering

The presence of odd-even parity states has encouraged us to study staggering effect for ^{154}Gd. Staggering pattern between the energies of the ground state and the (−ve) parity octupole bands have been calculated, ΔI = 1, using staggering function Eq. (13, 14) with the help of the available experimental data [29].

$$\text{Stag}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2) \quad (13)$$

with

$$\Delta E(I) = E(I+1) - E(I) \quad (14)$$

The calculated staggering pattern has illustrated in Figure 4. It shows an interaction between the (−ve) and (+ve) parity states.

4.6. Electric Monopole Transitions, Xif ′ f (E0/E2)

The electric monopole transitions, E0, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transition, Xif ′ f (E0/E2), [31] can be calculated using Eq. (15, 16) and results are presented in Table 3.

Table 3. *Calculated Xif ′ f (E0/E2)*

I_i^+	I_f^+	$I_{f'}^+$	*Xif ′ f (E0/E2)*$_{Exp*}$	*Xif ′ f (E0/E2)*$_{IBA}$
0_2	0_1	2_1	------	5.11
0_3	0_1	2_2	------	0.67
0_3	0_2	2_2	------	3.11
0_4	0_1	2_3	------	1.13
0_4	0_2	2_3	------	0.67
0_4	0_3	2_3	------	4.54
2_3	2_2	2_1	------	1.04
2_4	2_2	2_1	------	6.28
2_4	2_3	2_1	------	1.77

The small values of the Xif ′ f (E0/E2) indicate that the transition has small contribution of E0 transition, while the large valeue means that the decay of the state is mainly E0 which is by turn responsible for the shape of the nucleus.

$$Xif' f (E0/E2) = B(E0, I_i - I_f) / B(E2, I_i - I_{f'}) \quad (15)$$

Where

$$I_i = I_f = 0, I_{f'} = 2 \text{ or } I_i = I_f \neq 0, I_f = I_{f'}$$

$$Xif' f (E0/E2 = 2.54x10^9 A^{3/4} [E^5\gamma \text{ MeV}/ \Omega_{KL}]x\ \alpha (E2) \\ x[T_e(E0, I_i - I_f) / T_e(E2, I_i - I_{f'})] \quad (16)$$

A: mass number; depopulating it;

I_f: spin of the final state of E0 transition;

$I_{f'}$: spin of the final state of E2 transition;

Eγ: gamma ray energy;

Ω_{KL}: electronic factor for K, L shells [32];

α(E2): conversion coefficient of the E2 transition;

Te (E0, $I_i - I_f$): absolute transition probability of the E0 transition

between I_i and I_f states, and

Te (E2, $I_i - I_{f'}$): absolute transition probability of the E2 transition between I_i and $I_{f'}$ states.

5. Conclusions

IBA-1 model has been applied successfully to ^{154}Gd isotope and:

1. The levels energy are successfully reproduced;
2. The levels energy, transition probability ratios and potential energy surfaces are calculated and show X(5) characters to ^{154}Gd;
3. Electromagnetic transition rates B(E1) and B(E2) are calculated;
4. Forward bending has observed at angular momentum $I^+ = 6$;
5. Strength of the electric monopole transitions Xif ′ f (E0/E2) are calculated; and
6. Staggering effect has been calculated and beat pattern has obtained which show an interaction between the (−ve) and (+ve) parity states.

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